

A Reasoning Engine for the Gamification of Loop-Invariant Discovery

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Abstract. We describe the design and implementation of a reasoning engine that facilitates the gamification of loop-invariant discovery. Our reasoning engine enables students, computational agents and regular software engineers with no formal methods expertise to collaboratively prove interesting theorems about simple programs using browser-based, online games. Within an hour, players are able to specify and verify properties of programs that are beyond the capabilities of fully-automated tools. The hour limit includes the time for setting up the system, completing a short tutorial explaining game play and reasoning about simple imperative programs. Players are never required to understand formal proofs; they only provide insights by proposing invariants. The reasoning engine is responsible for managing and evaluating the proposed invariants, as well as generating actionable feedback.

Keywords: Program verification · Education · Loop invariants · Gamification · Theorem proving · Collaborative verification

1 Introduction

We introduce a reasoning engine that is the key enabling technology of IDG, the Invariant Discovery Game [28]. The game enables students and programmers without formal methods expertise to prove interesting statements about programs. Our reasoning engine does this by taking possibly incomplete or even incorrect insights about a program provided by students, programmers or other computational agents, combining such input with previous inputs, and giving users actionable feedback highlighting missing insights required to prove correctness.

An example of IDG is shown in Fig. 1. To reach this level of the game, players have to complete a tutorial that explains the interface. The game can be played at <http://invgame.atwalter.com> and the reader is encouraged to play along. The program under consideration is shown in the middle of the figure. The program's guarantee, $\text{cnt}^2 \leq n \ \& \ n < (\text{cnt}+1)^2$, is shown near the bottom. The goal of the player is to propose enough invariants to enable the reasoning engine to prove the guarantee. To help the player, the game provides feedback that includes a program state which satisfies the proposed invariants, fails the loop condition and falsifies the guarantee. Such a state should be unreachable

The screenshot shows the initial state of the game 'int square root'. On the left, a 'Program Trace' table shows the state of variables `n`, `sqr`, `odd`, and `cnt` for `n = 46`. The first row is highlighted in green, indicating it satisfies the invariant, while the rest are red. The main code editor shows the program logic with a `while` loop and a `Guarantee` box containing the expression `cnt^2 <= n & n < (cnt+1)^2`. A 'New Game' pop-up window is open, asking the player to propose invariants. At the bottom, there are 'Expression Reference' and 'Code Reference' toolbars.

n	sqr	odd	cnt
46	-	-	-
46	1	1	0
46	4	3	1
46	9	5	2
46	16	7	3
46	25	9	4
46	36	11	5
46	49	13	6

Fig. 1. The initial state of the game.

and to make progress, the player is asked to propose new invariants that rule out this state.

As shown in the figure, players can hover over technical terms such as *state*, resulting in a pop-up window explaining the term. Players can also generate program traces by providing inputs; in the figure a trace for `n = 46` is shown. This trace suggests that the variables `odd` and `cnt` can never be negative, leading to the player proposing `odd >= 1` and `cnt >= 0` in the expression box. The player also realizes that `odd` must be odd and proposes `odd % 2 = 1`, after consulting the expression reference located at the bottom of the screen. As the player types a proposed invariant, the game checks that it is satisfied by the current trace; rows that satisfy the invariant are colored green and rows that do not satisfy the invariant are colored red (not shown). Only when all rows are green is the player allowed to propose the invariant by clicking on the red `+` button.

IDG adds the three expressions to the list of inductive invariants and responds with the feedback shown in Fig. 2a, which includes a new state that the player should rule out by suggesting more invariants. At this point, the player generates a trace for `n = 3`, to compare an actual trace with the state shown by the game. After trying a few more traces (consider the trace in Fig. 1), the player recognizes that `sqr` is always the `cnt+1th` perfect square, and enters the expression `sqr = (cnt+1)^2`. IDG then adds this expression to the list of potential invariants and responds with the feedback shown in Fig. 2b. To explore why this is a potential invariant and not an inductive invariant, the player clicks on the `?` icon next to the potential invariant resulting in the feedback shown in Fig. 2c. Note that the first state satisfies the known inductive and potential invariants, but after executing the body of the loop once, we wind up with the second state, which does not satisfy the potential invariant. The problem is that the player

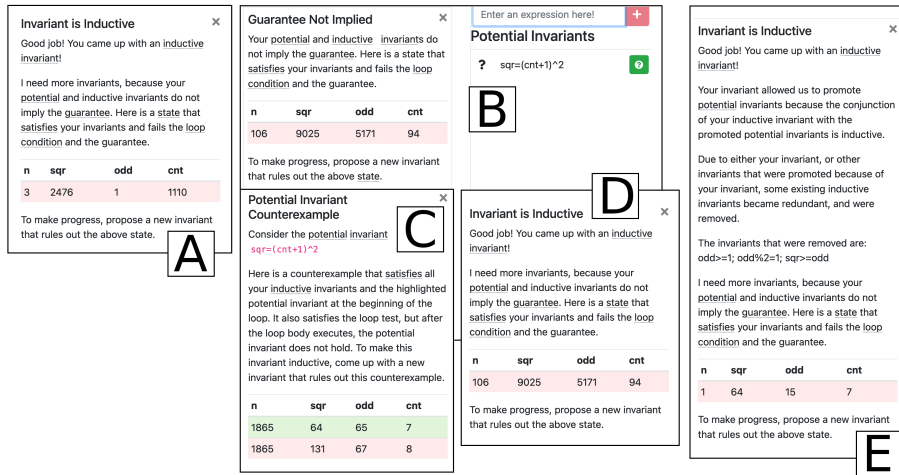


Fig. 2. Feedback generated in response to proposed invariants.

has not (yet) suitably constrained **odd**, which can never be greater than **sqr**, so the player proposes $sqr \geq odd$, which IDG proves to be inductive. IDG responds with the feedback shown in Fig. 2d. After looking at some traces, the player has the insight $odd = cnt * 2 + 1$. IDG confirms that this is an inductive invariant and responds with the feedback shown in Fig. 2e. Notice that the potential invariant was promoted to an inductive invariant. The promotion highlights one of the ways in which the reasoning engine manages and uses incomplete information. In addition, the reasoning engine also determined that three of the inductive invariants are now redundant and, in order to limit the cognitive load on users, these redundant invariants were removed. The reader is encouraged to finish the game, which only requires one more invariant.

Notice that players are only responsible for proposing invariants, not for any kind of formal reasoning. *Collaborative play* is also possible by combining proposed invariants from multiple players, some of whom are humans and some of whom are computational agents. In addition, the game has been used at Northeastern University to teach undergraduate students about loop invariants for imperative programs.

An evaluation of IDG performed using players from Amazon’s Mechanical Turk showed that IDG players were able to identify invariants that imply correctness for programs beyond the reach of fully automated systems and that IDG was more effective at eliciting such invariants than previous games [28].

The major contribution of this paper is the design of a reasoning engine that allows students and programmers without formal methods knowledge to collaboratively prove statements about programs which were not provable by best-in-class automated tools [7], with minimal training, in under one hour. We describe the core algorithms for taking incomplete proposed invariants from multiple sources and combining them to verify programs and to produce concrete

and actionable feedback enabling users to propose further invariants, without having to understand formal proofs. Feedback is designed to leverage the users' existing expertise in programming, *e.g.*, by showing program traces, exhibiting program states that need to be ruled out by new invariants and by highlighting relevant portions of the program code.

2 Related Work

There has been significant work in the area of program verification tools. We give a short overview of several classes of program verification tools and discuss their relevance to our work. We also discuss previous work in the area of gamification applied to program verification.

General-purpose interactive theorem provers are one class of such tools. Examples include ACL2, ACL2s, Agda, Coq, HOL, HOL-Light, Isabelle and PVS [5, 12, 16, 18, 20, 22, 25, 26]. In the hands of experts, many amazing theorems have been proved about complex systems using interactive theorem provers [1, 2, 8, 19, 21]. These tools can be thought of as proof checkers because users are responsible for sketching proofs and providing enough guidance so that the tools accept their proofs. Since program verification is an undecidable problem, to be fully general, current tools have to be interactive and one of the main goals in the area is to provide useful mechanisms that increase automation. For example in ACL2, there are dozens of ways of using and combining libraries, theorems, external tools, specialized reasoning engines, decision procedures, user-configured proof-search methods, etc. to program the theorem prover so that it can effectively reason about problems in a particular domain. While significant progress has been made, to effectively use such tools requires significant training over the course of several months and requires understanding the underlying language, logic, proof theory and features of the interactive theorem prover.

Fully automated theorem provers, such as Alt-Ergo, CVC, Inez and Z3 [4, 6, 14, 23] are able to automatically prove or disprove conjectures, but only for limited fragments of logic. Program verification tools like Spec#, VCC, and ESC/Java [3, 13, 17] allow programmers to annotate their programs with properties that an automated tool then attempts to prove hold. These tools have numerous options and significant learning curves.

Outside of the traditional sphere of formal methods research, research into the gamification of program verification and theorem proving is an emerging area of research. The DARPA Crowd Sourced Formal Verification (CSFV) program, launched in the early 2010s, produced games which target different kinds of formal verification problems [15]. Two of the games developed for the CSFV program are of particular interest: Xylem and Binary Fission.

In 2018, Bounov *et al.* developed a loop invariant discovery game in which, unlike many of the CSFV games, users are exposed to mathematical symbols and operators directly [7]. This game, called InvGame, is similar to many of the CSFV games in that it does not display the code being reasoned about. Bounov *et al.* found that, by crowdsourcing the game on Amazon Mechanical

Turk, players collectively were able to find the loop invariants needed to prove 10 of 14 benchmarks which leading automated program verification tools could not prove. Bounov *et al.* found that the decision not to abstract away the underlying math enabled players to use their existing mathematical expertise while playing the game. On the other hand, while the choice to not display program code reduced the amount of information players had to work with, this resulted in a game that presented a significantly lower cognitive burden for players. Bounov *et al.* theorize that this made the game easier for non-experts to play.

We developed the loop invariant discovery game IDG, discussed in the introduction, which shows code and does not abstract away the underlying mathematics of the loop invariants, thereby leveraging the existing programming expertise of players. The evaluation of IDG shows that it is the most effective invariant discovery game [28]. Quiring and Manolios have recently developed GACAL, a tool that can play the loop-invariant discovery game, outperforming most human players and providing an example of a computational agent that can augment human players [27].

3 System Architecture

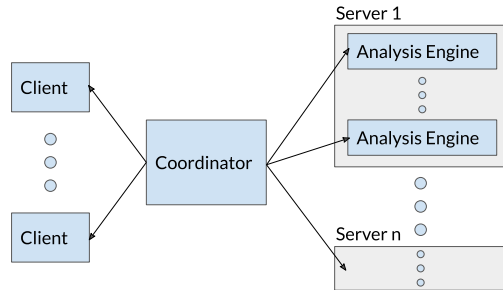


Fig. 3. System architecture for IDG

The architecture for IDG, shown in Fig. 3, consists of clients, a coordinator, and Invariant Analysis Engine (IAE) instances. Multiple clients and IAE instances are allowed, each of which may be on a separate computer. Players run the client, which handles the UI (User Interface) and game state, in their browser. When a player suggests an expression within the client, the client queries the coordinator, which selects an idle IAE instance and asks it to analyze the expression. When the IAE instance completes its analysis, the coordinator forwards the results to the client, which updates the UI and game state appropriately.

The client is responsible for storing the user’s game state, as well as handling the game UI. For each program the user has attempted, the client persists a record of the expressions the user proposed, even those which are not shown in

the UI. This allows the user to continue working on programs even after leaving the game. The client also computes user scores based on the expressions they submit.

The client performs local checks to prevent the user from submitting obviously useless expressions to the IAE. This includes ensuring that expressions are syntactically valid, different from previously submitted expressions and not falsified by the current trace. These checks are relatively simple and implementing them in the client, as opposed to having to engage the IAE, results in instantaneous user feedback.

The coordinator is responsible for choosing an IAE instance to handle a client query, as well as storing data logged by clients and maintaining a cache of the programs that users submit. The coordinator is intended to be low-overhead, with as much of the heavy computation limited to the client and the IAE instance. However, should multiple coordinators be necessary to handle the volume of queries being made by clients, the design of the coordinator does not preclude this.

The IAE, which is the focus of this paper, is described in Section 4.

4 Invariant Analysis Engine

The playthrough of IDG described in the introduction highlighted three different broad types of actions that the Invariant Analysis Engine (IAE) must perform: (1) trace generation, (2) expression characterization, handling and promotion and (3) counterexample and feedback generation.

The above actions are performed on arbitrary programs written in the Simple Imperative Programming language (SIP), whose abstract syntax is shown in Table 1. SIP expressions consist of Boolean and arithmetic expressions over unbounded integers, natural numbers, and rational numbers, as well as function calls. Executable statements include variable declarations, assignments, compositions, conditionals, while loops and print statements. SIP also includes assumption, assertion and **cassign** assignments: **cassign**($[vars]$, ϕ) nondeterministically assigns values to the variables in $vars$ that satisfy the expression ϕ . Function definitions optionally include **pre** and **post** statements, corresponding to pre- and post-conditions. The semantics of SIP programs are now standard.

Table 1. Abstract Syntax of SIP

$\langle r \rangle$ indicates that r is optional and $r \dots$ indicates 0 or more r 's.
 i identifier
 t type specifier (Boolean, Natural, Integer, Rational)
 e Boolean or numeric expression
 $s ::= \text{var } i:t \mid i:=e \mid s; s \mid \text{if}(e)\{s\}\text{else}\{s\} \mid \text{while}(\langle e \rangle)(e)\{s\} \mid \text{print}(e\dots) \mid$
 $\quad \text{assume}(e) \mid \text{assert}(e) \mid \text{claim}(e) \mid \text{cassign}([i\dots], e)$
 $f ::= \text{fn } i(i:t\dots):t \{ \langle \text{pre}(e); \rangle \langle \text{post}(e); \rangle s \}$
 $p ::= f \dots$

Our goal is to enable human and computational agents to collaboratively reason about SIP programs, but due to space limitations, in this paper we make the simplifying assumptions that SIP programs are well-typed and consist of a single function with a single while loop. All of the levels in IDG satisfy these restrictions.

We defined the syntax and semantics of SIP programs using the ACL2s theorem prover [9]. We implemented a trace generation capability that allows us to generate a trace, given a SIP program as input. Finally, we implemented a procedure that given such a SIP program, generates verification conditions for the loop invariant and the post condition (referred to as the guarantee in IDG).

4.1 Expression Characterizations

An IDG player’s goal is to submit expressions that progressively strengthen a loop invariant until it is strong enough to prove the property corresponding to the guarantee. To support collaborative and incremental reasoning, expressions are submitted incrementally, potentially from multiple sources, as the game is played. The IAE determines which proposed expressions provide useful information about the program under consideration, and which do not, by characterizing proposed expressions, as described below.

Let e be the expression under consideration. The characterization of e depends on T , the type information for the program under consideration. T depends only on the program and is not affected by game play. The characterization of e also depends on the history of expression characterizations that occurred during the game play before the proposal of e . Part of this history is recorded in I , the set of currently known inductive loop invariants, *i.e.*, I is a set of previously proposed expressions and $\bigwedge I$ is inductive. I depends on game play; initially I is empty, but as agents play the game, they propose expressions that may be added to I . We also have P , the set of currently known *potential invariants*: expressions with the potential to be characterized as inductive invariants after more game play. How this happens will be described shortly. Given this setup, we characterize e as follows.

(1) **type-tautology**: e is a tautology, assuming the program’s types, *i.e.*, $T \Rightarrow e$. For example $y - x \leq y$ is a type tautology if x is a natural number and y is an integer. In the sequel, we assume T implicitly as a hypothesis as appropriate.

(2) **non-inv**: e is not an invariant and the IAE has a counterexample c , an assignment to the input variables that satisfies T such that running the program with c results in a state at the location of the loop invariant that falsifies e . This state may occur when the loop is first reached, or after some number of loop iterations.

(3) **inductive**: e is inductive, *i.e.*, for all legal inputs, e holds when execution of the program first reaches the loop and for all states in which I , e and the loop test hold, then after executing the body of the loop, e holds. Notice the dependence on I , the set of currently known inductive loop invariants. Note that all **type-tautology** expressions are also **inductive** expressions, so the characterizations are not disjoint.

(4) **potential**: e is a potential invariant if it is not **non-inv** and it is not **inductive**. For example, if e is an invariant, but is not inductive given I , then e will be characterized as being potential. Notice the dependence on I . If I is expanded in future game play, it is possible that an element $p \in P$ becomes inductive, with respect to the expanded I , in which case we say that p can be *promoted*.

If e is inductive, we can add it to I and if it is potential, we can add it to P , but we have to be careful because any of these moves can have consequences, which lead to two more characterizations.

(5) **displaced**: e is a displaced invariant if it is implied by the existing set of invariants ($I \Rightarrow e$). Displaced invariants are redundant, as they provide no new information beyond what is already known, so they can be dismissed.

(6) **displaced-pot**: e is a displaced potential invariant if it is equivalent to some subset of potential invariants, assuming that the set of invariants hold. That is, $\langle \exists S \subseteq P :: I \Rightarrow (\bigwedge S = e) \rangle$. Displaced potential invariants provide no new information and also do not limit future promotions, so they can be dismissed. Not limiting future promotions is the reason why the definition of a displaced potential invariant is stricter than the definition of a displaced invariant. Due to space limitations, we ignore this characterization in the sequel.

We note that the characterization problem is undecidable, as we have to check the validity of formulas that include addition and multiplication, hence, our final characterization.

(7) **unknown**: When none of the above cases hold, we say that e is **unknown**.

4.2 Theorem Prover

The IAE depends on a theorem prover that provides the functionality we describe in this section. We use the ACL2s theorem prover, but any appropriate theorem prover can be used. However, we note that the **cgen** counterexample generation [10] and **defdata** data definition [11] features of ACL2s are key technologies in our implementation of the IAE.

The logic of the theorem prover must be expressive enough to support the Verification Conditions (VCs) generated by the IAE, which include implicitly universally quantified formulas over the SIP-supported types (Booleans, natural numbers, integer and rationals) and operators (arithmetic and Boolean). The characterizations from the previous section are examples of the kinds of queries the IAE generates.

Given a query, the theorem prover either returns the constant **proved**, if it can prove that the formula holds, or it returns counterexamples disproving the query, or **unknown** is returned. Due to undecidability, we cannot rule out the last case.

The IAE interacts with the underlying theorem prover through a single procedure, **GENCHKVCs**. This procedure generates VCs for the given SIP program, using the verification condition generation capability mentioned above, and submits the VCs to the underlying theorem prover. In more detail, **GENCHKVCs** takes as input a SIP program, p , and a sequence of SIP statements, t , over the

1: structure InvariantState	
2: Invs	▷ inductive loop invariants
3: Plnvs	▷ potential invariants
4: structure Program	
5: Test	▷ The test for the loop
6: Body	▷ The body of the loop

Fig. 4. Type definitions for IAE

variables in p , with exactly one **assert** statement that must be the last element of t . The procedure generates verification conditions that check whether the assertion in t holds, under the assumption that the variables appearing in t have the types specified in p . Finally, we implemented a procedure that given such a SIP program, generates verification conditions for the loop invariant and the post condition (referred to as the guarantee in IDG).

If an algorithm listed here does not explicitly handle the **unknown** case when proving a VC, assume that the IAE will notify the client that it couldn't generate any useful feedback for the user.

4.3 Core IAE Algorithms

We now present the core IAE algorithms. We use the notation **alias** $foo := bar.baz$ inside of algorithms to mean that foo is just another name for $bar.baz$, *i.e.*, assigning a value to foo will also modify the value of $bar.baz$, and vice versa.

The data structures used to store expressions, expression states, and programs are defined in Fig. 4. For presentation purposes, only a small subset of the data structures and fields in the IAE are shown, *e.g.*, the IAE includes data structures and fields to support the recording of all queries and characterization results, which are useful for evaluating gameplay mechanics. In addition, we have simplified what actually happens, *e.g.*, in this paper we only consider programs with a single while loop, which significantly simplifies matters. In the actual implementation, we have a summary-based approach that supports reasoning about arbitrary SIP programs.

Commonly occurring arguments to the procedures defined in this section include: (1) p , the type-checked SIP program under consideration, (2) s , a structure encapsulating the current state of gameplay as described above and (3) e , the expression under consideration, which is a Boolean expression over program variables. If a set of expressions is used in a Boolean context, then it is implicitly conjoined.

Table 2 contains information about the procedures that are used in the description of the IAE algorithms below and is useful as a reference for understanding the algorithms in this paper.

The procedure for evaluating proposed loop invariants is shown in Fig. 5. The procedure takes as input (1) p , the program under consideration, (2) s , the structure described above, which includes the set of inductive and potential invariants and (3) e , the proposed invariant. It returns a tuple $(kind, res)$, where $kind$ is

Table 2. IAE Procedure Listing

GENCHKVCS (p, g)	Generate VCs that the SIP statement g gives rise to, with type information from program p . Returns proved if VCs are proved, a counterexample if one was found and unknown otherwise.
PROPOSELOOPINV (p, s, e)	Characterizes e and updates any other expression characterizations affected. Returns ($kind, res, s$) where $kind$ is the expression characterization for e , res is information that may be used for feedback, and s is the updated game state.
HASCTRXS(c)	Returns true if c corresponds to a theorem prover query that found counterexamples and false otherwise.
GENTRACE(p, a)	Generates a trace of p using variable assignments from a .
CHECKUPTOLOOP (p, e)	Generates subprogram of p up to, but not including the loop, ending with a statement asserting e .
CHECKLOOP (p, e)	Generates subprogram of p up to the loop, which is modified to allow execution to nondeterministically terminate early, and followed by a statement asserting e .
PROMOTE(p, s)	Promotes as many potential invariants as possible to inductive invariants, for program p and state s .
REMDISPLACED(s)	Removes displaced inductive invariants from s .

the characterization of e and res includes extra information when the characterization is **non-inv**: this extra information includes a trace that can be used to show the player why e is not an invariant. The procedure also updates s ; this happens in the call to procedure PROMOTE and only when the characterization of e is **inductive** or **potential**.

PROPOSELOOPINV starts by checking whether e is a **type-tautology**, by querying the theorem prover. The call to GENCHKVCS will generate a logical formula stating that e follows from the just the type information in program p . Notice that in this call to GENCHKVCS, we have satisfied the preconditions for the procedure as the second argument is a sequence of SIP statements over the variables in p , with exactly one **assert** statement, which is the last statement in the sequence. We will leave such checks to the reader in the sequel. The next check is whether e is displaced, which corresponds to checking if e holds, assuming I , the set of invariants and the type information in program p . Next, we check if e holds the first time program execution reaches the loop, using procedure CHECKUPTOLOOP, which takes as input a program, p , and an expression, e , and generates a sequence of SIP statements similar to p . The sequence includes all statements in p up to, but not including the loop. Variable declarations are also not included. Lastly, we add a final statement asserting e . If we find counterexamples, then clearly e is not a loop invariant. If the theorem prover returns **unknown**, then we return. Otherwise, we know that e holds when program execution reaches the loop for the first time. Notice that it is still possible for e to not be an invariant, and the next check is our final attempt to check this. We check if e is a **non-inv**, using the procedure CHECKLOOP, which takes as input a program, p , and an expression, e , and generates a se-

```

1: procedure PROPOSELOOPINV( $p, s, e$ )
  ▷ Returns a tuple ( $kind, res$ )
2: alias  $I := s.Invs$ 
3: alias  $P := s.PInvs$ 
4:  $tautology := \text{GENCHKVCS}(p, \text{assert}(e));$ 
5: if  $tautology = \text{proved}$  then
6:   return (type-tautology, nil)
7:  $displaced := \text{GENCHKVCS}(p, \text{assume}(I); \text{assert}(e));$ 
8: if  $displaced = \text{proved}$  then
9:   return (displaced, nil)
10:  $eChk := \text{GENCHKVCS}(p, \text{CHECKUPTOLOOP}(p, e))$ 
11: if  $\text{HASCTRXS}(eChk)$  then
12:    $trace := \text{GENTRACE}(p, eChk)$ 
13:   return (non-inv, ( $eChk, trace$ ))
14: if  $eChk = \text{unknown}$  then
15:   return (unknown, nil)
16:  $eChk := \text{GENCHKVCS}(p, \text{CHECKLOOP}(p, e))$ 
17: if  $\text{HASCTRXS}(eChk)$  then
18:    $trace := \text{GENTRACE}(p, eChk)$ 
19:   return (non-inv, ( $eChk, trace$ ))
20:  $P := P \cup \{e\}$ 
21:  $\text{PROMOTE}(p, s)$ 
22: if  $e \in I$  then
23:   return (inductive, nil)
24: else
25:   return (potential, nil)

```

Fig. 5. ProposeLoopInv

quence of SIP statements similar to p . The sequence includes all statements in p up to the loop, but not including variable declarations. In addition, the loop is modified to allow execution to nondeterministically terminate early. Lastly, we add a final statement asserting e . ACL2s allows us to express such formulas, using recursion, but if the underlying theorem prover is not expressive enough, it would be fine to either skip this test or to traverse the loop no more than some constant number of times. If no counterexamples are found, then we will treat e as a potential invariant, by adding it to P and the PROMOTE procedure will be called to check if e or any other element of P can be promoted. We note that it is possible that $I \wedge e$ is not inductive, but for e and other expressions in P to be promoted anyway.

The procedure for promotion is shown in Fig. 6. The procedure takes as input p , the program under consideration and s , the structure described above, which includes the set of inductive and potential invariants. It modifies s by updating the potential and inductive invariants in s .

PROMOTE starts with some simple assignments, including initializing X to be a copy of P , the potential invariants in s . The outer **repeat** loop is taken as

```

1: procedure PROMOTE( $p, s$ )
    $\triangleright$  Promotes PInvs and updates  $s$ 
2: alias  $P := s.PInvs$ 
3: alias  $I := s.SInvs$ 
4:  $lt := p.test$ 
5:  $lb := p.body$ 
6:  $X := P$ 
7: repeat
8:    $Done := \mathbf{true}$ 
9:   for  $x \in X$  do
10:     $g := \mathbf{assume}(I \wedge X \wedge lt); lb; \mathbf{assert}(x);$ 
11:    if GENCHKVCS( $p, g$ )  $\neq$  proved then
12:       $X := X \setminus \{x\}$ 
13:       $Done := \mathbf{false}$ 
14: until  $Done$ 
15:  $I := I \cup X$ 
16: REMDISPLACED( $s$ )
17:  $P := P \setminus X$ 

```

Fig. 6. Promote

long as an element is removed from X , which can only occur $|X|$ times. Each iteration of the **repeat** loop has an inner **for** loop that can potentially remove elements from X . The key insight is that an element x of X is not inductive if it does not hold after the body of the loop, assuming that I, X and the loop test hold. This check is repeated until we reach a fixpoint. That is, when the **repeat** loop finishes, we know that assuming I, X and the loop test all hold, then after execution of the loop body, I, X hold. That I holds follows from the assumption that $s.PInvs$ is a set of loop invariants, so I holds after the loop with the weaker hypothesis that I holds before the loop. To see that X holds after the loop, suppose not, then there is some $x \in X$ which does not hold and it would have been removed by the **for** loop. Notice that X is the largest subset of P which can be promoted. The final value of X is determined by a greatest fixpoint computation. Note that if we use a least fixpoint computation, by adding elements of P to I one at a time, every element so promoted is really an invariant, but we are not guaranteed to have promoted all the potential invariants that are promotable. The procedure REMDISPLACED maintains the invariant that I does not include any invariant that is implied by the rest of the invariants. When we update I by adding X to it, we open up the possibility that I contains displaced invariants, *e.g.*, it is possible that X contains a strong invariant that strictly implies multiple invariants in I . Hence, REMDISPLACED takes s as input and removes displaced invariants from I . As mentioned previously, IAE also performs displacement for potential invariants, which is a more complicated process.

4.4 IAE Generated Feedback

The IAE provides useful feedback to the user. We have already seen how this works if the proposed invariant is characterized as a tautology, a displaced invariant or a non-invariant. If neither of the cases hold, then the IAE check whether the IDG level has been *solved*: the program’s guarantee holds when program execution reaches the guarantee starting from any state that satisfies all of the known inductive invariants, the negation of the loop test and executes all the code following the loop.

If any promotion occurs, then the IAE will generate a query to check if the level has been solved. If so, the user is congratulated and is able to move on to another level. If not, any generated counterexamples are recorded.

If the level has not been solved, then the IAE will generate a query just like the solved query, except starting states are assumed to satisfy all of the known inductive *and potential* invariants. If a counterexample to this query is found, the IAE will return it to the user and the user will be asked to propose an invariant that rules out this state. Notice that such a counterexample illustrates that more invariants are needed, even if we consider the potential invariants to be inductive. If no counterexample is found to this query, the IAE will use the counterexamples generated from the solved query.

The user is also able to request interesting counterexamples, including counterexamples to the inductivity of potential invariants.

5 Evaluation

In order to evaluate the effectiveness of the reasoning engine, we recruited 300 players from Amazon’s Mechanical Turk platform and randomly assigned them to play IDG and IDG-T, a game based on InvGame [7], the state-of-the-art in invariant discovery games. IDG-T does not show code; it only shows traces. We implemented IDG-T, instead of using data from InvGame [7], in order to limit confounding variables, such as differences in player populations, user interface elements, definitions, metrics and statistical tests.

In a previous paper, we evaluated the gamification aspects of the data collected, which included the experimental setup, the provenance of the benchmark problems used, information on the participants, a comparison of levels solved, cheesability considerations, player skill data and player feedback [28]. A very brief, incomplete, summary of the previous evaluation is provided. The games consisted of twelve levels. For eleven of these levels, there was at least one IDG player who was able to (individually) solve the level. For IDG-T, only seven of the levels were solved individually. However, if all the proposed invariants by all the IDG-T players are collected, then a total of nine levels were solved. The level that was not solved by IDG players was also not solved by IDG-T players.

For all analyses below, we only consider data for users who completed at least one level. The first level players were asked to solve was exactly the same as the level used in the tutorial, hence we used this criterion to remove players

who were gaming the game. For all of the experiments reported, we identified the metrics and p-values (0.05) that we consider statistically significant before running the analyses.

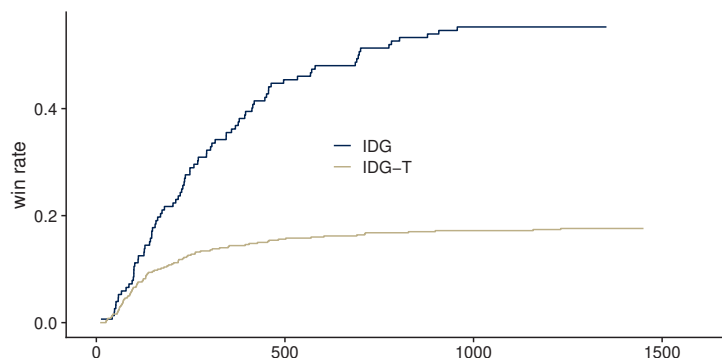


Fig. 7. The win rate for IDG and IDG-T versus active play time, in seconds.

In Figure 7, we show the *win rate* of players for both games. The *x*-axis corresponds to time in seconds and the *y*-axis is the win rate of the players. Each player attempted some number of levels, typically six; the attempted levels for a game is the sum of the attempted levels over all the players of the game. The *win rate* of a game at time t is the ratio of levels solved to levels attempted, after t seconds of game play. Notice that the numerator in the ratio depends on t , but the denominator does not, *e.g.*, if at time 500, IDG players solved 1,000 levels out of a total of 2,000 attempted levels, then the win rate, at time 500, is 0.5. What the graph shows is that at 1,500 seconds, the IDG win rate is 0.553, while the IDG-T win rate is only 0.176. The win rate allows us to estimate the expected number of levels players of the games will solve within a certain amount of time. As the figure clearly shows, after any non-trivial amount of gameplay, IDG players solve more levels than IDG-T players.

Care was taken to deal with outliers. For example, we noticed that certain players loaded levels, but did not attempt to solve them. Therefore, only players who proposed an expression for a level and had a total active level play time of more than 10 seconds were considered to have attempted a level. No level was solved in less than 20 seconds. We also noticed that four players spent more than 1,500 seconds on a level, but when we looked at the logs, there were frequent periods of inactivity, indicating that they were multitasking or distracted, hence we ignored these players. The computation of active level play time is somewhat complex. If a user played the same level in multiple browser sessions, we include time spent on all plays and added a 30 second penalty for each session. We ignored periods of inactivity that lasted at least 5 minutes, but added a 2 minute penalty for each such period. This allows us to better approximate the actual

active time spent by players, *e.g.*, there were players with total, unadjusted play time on a single level of over 4,000 seconds, almost all of which was idle time.

We use the Mann-Whitney U test to compare the win rates over time for IDG and IDG-T. The test indicates a statistically significant difference between the two curves ($p = 1.24e-30$). The win rate was partly motivated by cumulative incidence, from the field of survival analysis [24].

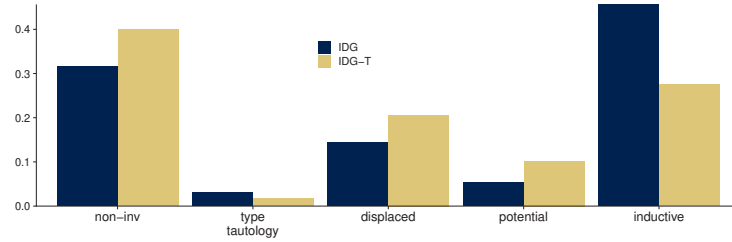


Fig. 8. Distribution of the types of submitted expressions, in aggregate. Types are ordered from left to right from least useful (non-inv) to most useful (inductive).

Fig. 8 sheds light on why IDG players outperformed IDG-T players: IDG players submitted significantly more useful expressions (inductive and potential invariants) than IDG-T players. This was due to the feedback generated by the reasoning engine. A Pearson’s Chi-Squared test with Yates’ continuity correction indicates a statistically significant correlation between game variant and the distribution of expression types submitted ($p = 2.97e-24$, Cramer’s $V = 0.16$).

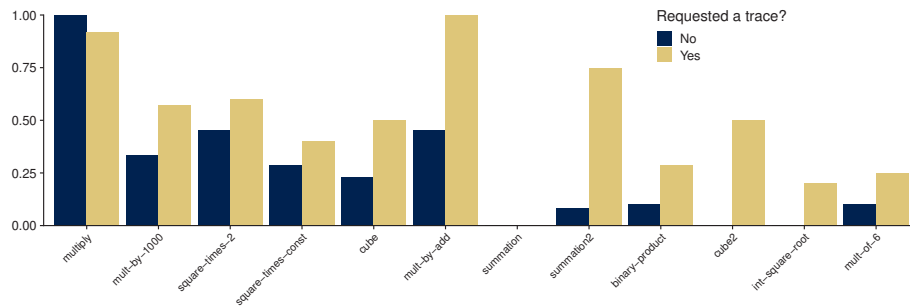


Fig. 9. Prove ratio for IDG players by level, stratified by whether the player requested a trace when playing the level.

Fig. 9 shows how the *prove ratio* for IDG players, the proportion of players who proved a level to those who attempted it, differs based on whether players requested a trace or not. All shown ratios have a non-zero denominator, *i.e.*, for every attempted level, there was at least one player who requested a trace and

one who did not. IDG provides the player with a trace when the level is loaded, and may provide either single states or snippets of traces in its feedback, but the ability to generate custom traces for given input values seems to be quite helpful for discovering invariants. A Pearson's Chi-Squared test with Yates' continuity correction indicates a statistically significant correlation between whether a participant proved a level and whether they requested a trace ($p = 0.046$, Cramer's $V = 0.14$). Additionally, this feature was used often, as more than 29% of all IDG level attempts used it.

Our evaluation shows that IDG is more effective than IDG-T at eliciting useful invariants from players. The key enabling technology for IDG is the reasoning engine.

6 Future Work

There are many interesting research directions to pursue, some of which we outline in this section. One idea is to develop more educational games that can be used to introduce program verification to students. We already use IDG at Northeastern University to introduce verification condition generation and reasoning about imperative programs to undergraduate students. We would like to create games that support more complicated programs, using richer languages and moving towards the analysis of industrially-relevant programs. Also, we would like to provide greater support for crowdsourced program verification, more visualization capabilities and more customizable games. Finally, we believe that there are many opportunities to develop computational agents that can augment and complement human cognitive abilities. The GACAL system is an early example [27].

7 Conclusion

We have described the design and implementation of an interactive reasoning engine for loop-invariant discovery games that enables regular programmers without formal methods training to collaboratively prove program correctness of simple imperative programs. The games are Web-based, requiring only a browser to use. After a ten minute tutorial, players with no formal methods expertise are able to specify and check loop invariants. Our reasoning engine provides actionable feedback in the form of program states that players are asked to rule out. This feedback enables players to effectively use their programming insights to propose invariants and check invariants. A detailed evaluation has shown that our game is more effective than existing games in soliciting useful expressions from players and in helping players prove program correctness.

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